Comment on "Quantum entangled dark solitons formed by ultracold atoms in optical lattices"

Jacek Dziarmaga,[1] Piotr Deuar,[2] and Krzysztof Sacha[1]

[1]Instytut Fizyki im. Mariana Smoluchowskiego and Mark Kac Complex Systems Research Center, Uniwersytet Jagielloński, ul. Reymonta 4, PL-30-059 Kraków, Poland

[2]Institute of Physics, Polish Academy of Sciences, Al. Lotników 32/46, 02-668 Warsaw, Poland

The recent Letter [1] describes full quantum simulations of a dark soliton in a Bose-Einstein condensate in a regime where the system cannot be described by the perturbative approach. The authors argue, based on the filling in of the two-point correlator $g^{(2)}$, that a photograph of a condensate would reveal a smooth atomic density without any localized dark soliton. This is in contrast to the perturbative regime where a photograph would show a dark soliton with a random position [2]. While we admire the quantum simulations and other results in [1], we think that their conclusion about the outcome of a single experiment is not justified by this property of $g^{(2)}$.

As the question of whether and when dark solitons fill in in single realisations has been a recurring and sometimes confusing one in the field, it is important to pin down what conclusions can or cannot be made. With this aim, we provide the following counterexample in the non-perturbative regime (see also [3]).

Let $\phi_q(x) \propto \tanh[(x-q)/\xi]$ be a standard condensate wave function with a dark soliton at q. With $\hat{a}_q = \int dx\ \phi_q^*(x)\hat{\Psi}(x)$, a state $\left(\hat{a}_q^\dagger\right)^N|0\rangle$ is a condensate with a soliton at q, and let the N-particle state be a superposition of condensates with different q

$$|\psi_0\rangle \propto \int dq\ \psi_0(q)\ \left(\hat{a}_q^\dagger\right)^N|0\rangle\ , \tag{1}$$

where $\psi_0(q)$ defines the superposition.

After measurements of n atomic positions $x_1, ..., x_n$ the state (1) collapses to a conditional state

$$|\psi_n\rangle\ \propto\ \hat{\Psi}(x_n)..\hat{\Psi}(x_1)|\psi_0\rangle\ \propto\ \int dq\ \psi_n(q)\ \left(\hat{a}_q^\dagger\right)^{N-n}|0\rangle\ ,$$

where $\psi_n(q) = \phi_q(x_n)..\phi_q(x_1)\psi_0(q)$. The $(n+1)$-st measurement will find a particle at x_{n+1} with a probability

$$p_{n+1}(x_{n+1}) \propto \langle\psi_n|\hat{\Psi}^\dagger(x_{n+1})\hat{\Psi}(x_{n+1})|\psi_n\rangle.$$

This is equivalent to simultaneous measurement of all x_i.

Using the methods of [2], we simulated measurement of all $N = 5000$ particles on a lattice of 31 sites, where $x, q \in \{-15, 15\}$, assuming a soliton width $\xi = 1.5$, and a delocalized (uniform) superposition $\psi_0(q) \propto 1$ that is non-perturbatively wider than the soliton width. The inset in the figure shows the ensemble average particle density $p_1(x)$ and the main figure shows a generic histogram of particle positions $x_1, .., x_N$ measured in a single realization. Each single realization of the experiment finds a soliton localized at some definite but random q.

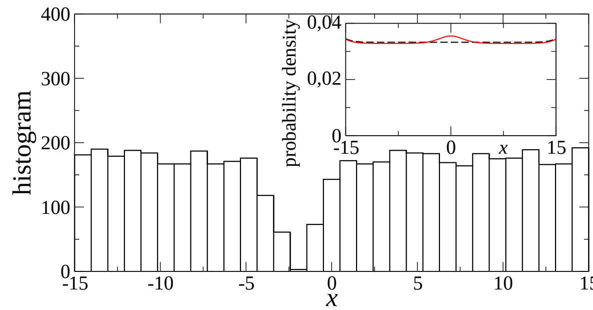

FIG. 1: (Color online) Histogram of measured atom positions in a single experiment from (1). Inset: single particle density $p_1(x)$ (dashed black line) and $g_2(x)$ (solid red line), normalized so that $\int g_2(x)dx = 1$.

What about the two-point correlator $g_2(x) = \langle\psi_0|\hat{\Psi}^\dagger(0)\hat{\Psi}^\dagger(x)\hat{\Psi}(x)\hat{\Psi}(0)|\psi_0\rangle$ that is analyzed in [1]? This turns out nearly uniform (see the inset). Hence, the "filled in g_2" → "filled in soliton" line of reasoning is clearly incorrect. Of course, the soliton simulated in [1] may still be greying, but the point here is that one cannot answer such a question by analyzing $g_2(x)$.

Our example demonstrates that, in some cases, a low order correlator like $g_2(x)$ is insufficient to draw conclusions on the outcome of a single experiment, and that the soliton in a BEC is such a case. Here, g_2 is equal to our $p_2(x_2)$ after the first particle was measured at $x_1 = 0$. A measurement of only one particle is not enough to collapse the soliton position. If we want to infer the soliton position from a histogram of particle positions, then the number of measured particles must be large enough to provide a histogram with a well-resolved soliton notch. This is a fundamental requirement for an accurate Bayesian inference of the soliton position.

Support within Polish Government scientific funds (2008-2011 KS, 2009-2012 JD) as a research project is acknowledged.

[1] R. V. Mishmash and L. D. Carr, Phys. Rev. Lett. **103**, 140403 (2009); see also: R. V. Mishmash *et al.*, Phys. Rev. A **80**, 053612 (2009).

[2] J. Dziarmaga and K. Sacha, J. Phys. B **39**, 57 (2006); Phys. Rev. A **66**, 043620 (2002); J. Dziarmaga *et al.*, J. Phys. B **36**, 1217 (2003); Phys. Rev. A **66**, 043615 (2002).

[3] J. Dziarmaga, Phys. Rev. A **70**, 063616 (2004).